Homogenous equation

1. $x\frac{dy}{dx} + \frac{y^2}{x} = y$

$\Rightarrow x\frac{dy}{dx} = y - \frac{y^2}{x} = \frac{xy - y^2}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{xy - y^2}{x^2}$

Put $y = vx$ ...

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{vx^2 - v^2x^2}{x^2} = \frac{v(1-v)x^2}{x^2}$

$\Rightarrow x\frac{dv}{dx} = v - v^2 - v$

$\Rightarrow \frac{dv}{+v^2} = -\frac{dx}{x}$

$\Rightarrow \int \frac{dv}{v^2} = -\int \frac{dx}{x}$

$\Rightarrow \frac{-1}{v} = +\log x + c$

$\Rightarrow -\frac{x}{y} = \log x + c$

$\Rightarrow \frac{x}{y} = \log x - \log c = \log \frac{x}{c}$

$\Rightarrow \frac{x}{c} = e^{x/y}$

$\Rightarrow x = c e^{x/y}$ Ans

Ex – 3A

1. $(x^3+y^3)dx - xy^2dy = 0$

$\Rightarrow (x^3+y^3)dx = xy^2dy$

$\Rightarrow \frac{dy}{dx} = \frac{x^3+y^3}{xy^2}$

Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{x^3+v^3x^3}{v^2x^3} = \frac{1+v^3}{v^2}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+v^3}{v^2} - v = \frac{1+v^3 - v^3}{v^2} = \frac{1}{v^2}$

$\Rightarrow \int v^2 . dv = \int \frac{dx}{x}$

$\Rightarrow \frac{v^3}{3} + C = \log x$

$\Rightarrow \frac{y^3}{3x^3} + C = \log x$ Ans

2. $xdy - ydx + \sqrt{x^2+y^2}\,dx = 0$

$\Rightarrow xdy + (\sqrt{x^2+y^2} - y)dx = 0$

$\Rightarrow xdy = (y - \sqrt{x^2+y^2})dx$

$\Rightarrow \frac{dy}{dx} = \frac{y - \sqrt{x^2+y^2}}{x}$

Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\Rightarrow v+x\frac{dv}{dx} = \frac{vx-\sqrt{x^2+v^2x^2}}{x} = \frac{vx-x\sqrt{1+v^2}}{x}$$

$$\Rightarrow v+x\frac{dv}{dx} = v-\sqrt{1+v^2}$$

$$\Rightarrow x\frac{dv}{dx} = v-\sqrt{1+v^2}-v$$

$$\Rightarrow \int\frac{dv}{\sqrt{1+v^2}} = -\int\frac{dx}{x}$$

$$\Rightarrow \log|v+\sqrt{1+v^2}| = -\log x + c$$

$$\Rightarrow \log x + \log|v+\sqrt{1+v^2}| = c$$

$$\Rightarrow \log x + \log\left|\frac{y}{x}+\frac{\sqrt{x^2+y^2}}{x}\right| = c$$

$$\Rightarrow \cancel{\log x} + \log|y+\sqrt{x^2+y^2}| - \cancel{\log x} = c$$

$$\Rightarrow \log|y+\sqrt{x^2+y^2}| = c \quad \text{Ans}$$

3. $ydx - xdy = \sqrt{x^2-y^2}\,dx$

$$\Rightarrow (y-\sqrt{x^2-y^2})\,dx = x\,dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{y-\sqrt{x^2-y^2}}{x}$$

Put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v+x\frac{dv}{dx}$$

$$\Rightarrow v+x\frac{dv}{dx} = \frac{vx-\sqrt{x^2-v^2x^2}}{x} = \frac{vx-x\sqrt{1-v^2}}{x}$$

$$= v-\sqrt{1-v^2}$$

$\Rightarrow x\frac{dv}{dx} = v - \sqrt{1-v^2} - v$

$\Rightarrow \int \frac{dv}{\sqrt{1-v^2}} = -\int \frac{dx}{x}$

$\Rightarrow \sin^{-1}(v) = -\log x + c$

$\Rightarrow \log x + \sin^{-1}\left(\frac{y}{x}\right) = c$ Ans

4. $2xy\frac{dy}{dx} = x^2+y^2$

$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2vx^2} = \frac{1+v^2}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v = \frac{1+v^2-2v^2}{2v} = \frac{1-v^2}{2v}$

$\Rightarrow \int \frac{2v}{1-v^2}dv = \int \frac{dx}{x}$

$\Rightarrow \log(1-v^2) = \log x + c$

$\Rightarrow \log\left(1-\frac{y^2}{x^2}\right) = \log x + C$

$\Rightarrow \log(x^2-y^2) + \log K = \log x$

$\Rightarrow \log K(x^2-y^2) = \log x$

$\Rightarrow x = K(x^2-y^2)$ Ans

5. $\frac{dy}{dx} = \frac{x^2+y^2}{2x^2}$

Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2x^2} = \frac{1+v^2}{2}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2} - v = \frac{1+v^2-2v}{2}$

$\Rightarrow \frac{dv}{v^2-2v+1} = \frac{1}{2}\cdot\frac{dx}{x}$

$\Rightarrow \int \frac{dv}{(v-1)^2} = \frac{1}{2}\int \frac{dx}{x}$

$\Rightarrow \frac{-2}{v-1} = \frac{1}{2}\cdot\log x + C$

$\Rightarrow \frac{-2}{\left(\frac{y}{x}-1\right)} = \frac{1}{2}\cdot\log x + c$

$\Rightarrow \frac{-2x}{y-x} = \log x + C$

$\Rightarrow \frac{2x}{x-y} = \log x + C$

$\Rightarrow 2x = (x-y)(\log x + C)$ Ans

6. $(x^2-y^2)dx - xy\,dy = 0$

$\Rightarrow (x^2-y^2)dx = xy\,dy$

$\Rightarrow \frac{dy}{dx} = \frac{x^2-y^2}{xy}$

Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2 - v^2x^2}{vx^2} = \frac{1-v^2}{v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1-v^2}{v} - v = \frac{1-v^2-v^2}{v} = \frac{1-2v^2}{v}$

$\Rightarrow \frac{v\cdot dv}{1-2v^2} = \frac{dx}{x}$

$\Rightarrow \frac{-1}{4}\int \frac{-4v\cdot dv}{1-2v^2} = \int \frac{dx}{x}$

$\Rightarrow \frac{-1}{4}\log(1-2v^2) + K = \log x$

$\Rightarrow \frac{-1}{4}\log\left(1 - \frac{2y^2}{x^2}\right) + K = \log x$

$\Rightarrow \frac{-1}{4}\log\left(\frac{x^2-2y^2}{x^2}\right) + K = \log x$

$\Rightarrow \frac{-1}{4}\{\log(x^2-2y^2) - 2\log x\} + K = \log x$

$\Rightarrow -\log(x^2-2y^2)^{1/4} + \frac{1}{2}\log x + K = \log x$

$\Rightarrow -\log(x^2-2y^2)^{1/4} + K = \frac{1}{2}\log x$

$\Rightarrow -\log c = \log\sqrt{x} + \log(x^2-2y^2)^{1/4}$ $[k = -\log c]$

$\Rightarrow -\log c = \log\sqrt{x}(x^2-2y^2)^{1/4}$

$\Rightarrow 0 = \log\sqrt{x}(x^2-2y^2)^{1/4} + \log c$

$\Rightarrow \log 1 = \log c\sqrt{x}(x^2-2y^2)^{1/4}$

$\Rightarrow 1 = c\sqrt{x}(x^2-2y^2)$ Ans

7. $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$

$\Rightarrow y^2 = (xy-x^2)\frac{dy}{dx}$

$\Rightarrow \frac{y^2}{xy-x^2} = \frac{dy}{dx}$

$\Rightarrow \frac{dx}{dy} = \frac{xy-x^2}{y^2}$

Put $x = vy$

$\Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

$\Rightarrow v + y\frac{dv}{dy} = \frac{vy^2 - v^2y^2}{y^2} = v - v^2$

$\Rightarrow y\frac{dv}{dy} = v - v^2 - v$

$\Rightarrow \frac{dv}{v^2} = -\frac{dy}{y}$

$\Rightarrow \frac{-1}{v} = -\log y + \log c$

$\Rightarrow \frac{y}{x} = \log cy$

$\Rightarrow y = x \log cy$ Ans

8. $(x^2+y^2)dy = xy\,dx$

$\Rightarrow \frac{x^2+y^2}{xy} = \frac{dx}{dy}$

Put $x = vy$

$\Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

$\Rightarrow v + y.\frac{dv}{dy} = \frac{v^2y^2+y^2}{vy^2} = \frac{v^2+1}{v}$

$\Rightarrow y\frac{dv}{dy} = \frac{v^2+1}{v} - v$

$\Rightarrow y\frac{dv}{dy} = \frac{v^2+1-v^2}{v}$

$\Rightarrow y.\frac{dv}{dy} = \frac{1}{v}$

$\Rightarrow v.dv = \frac{dy}{y}$

$\Rightarrow \int v.dv = \int \frac{dy}{y}$

$\Rightarrow \frac{v^2}{2} = \log y - \log c$

$\Rightarrow \frac{x^2}{2y^2} = \log \frac{y}{c}$ Ans

Q. $(x^2+y^2)dx - 2xy\,dy = 0$

$\Rightarrow (x^2+y^2)dx = 2xy\,dy$

$\Rightarrow \frac{x^2+y^2}{2xy} = \frac{dy}{dx}$

Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2vx^2} = \frac{1+v^2}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2-2v^2}{2v} = \frac{1-v^2}{2v}$

$\Rightarrow \frac{2v\,dv}{1-v^2} = \frac{dx}{x}$

$\Rightarrow -\int \frac{-2v\,dv}{1-v^2} = \int \frac{dx}{x}$

$\Rightarrow -\log(1-v^2) = \log x$

$\Rightarrow -\log\left(1-\frac{y^2}{x^2}\right) = \log x + \log c$

$\Rightarrow -\log(x^2-y^2) + 2\log x = \log x + \log c$

$\Rightarrow 2\log x - \log x = \log(x^2-y^2) + \log c$

$\Rightarrow \log x = \log c(x^2-y^2)$

$\Rightarrow x = c(x^2-y^2)$ Ans

10) $\frac{dy}{dx} = \frac{y(y+x)}{x(y-x)}$

Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{vx(vx+x)}{x(vx-x)} = \frac{v\cdot(v+1)x}{(v-1)x}$

$\Rightarrow x\frac{dv}{dx} = \frac{v^2+v}{v-1} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{x^2+v-x^2+v}{v-1} = \frac{2v}{v-1}$

$\Rightarrow \frac{v-1}{2v}\cdot dv = \frac{dx}{x}$

$\Rightarrow \frac{1}{2}\int dv - \frac{1}{2}\int \frac{dv}{v} = \int dx$

$\Rightarrow \frac{1}{2}v - \frac{1}{2}\log v = \log x - \log c$

$\Rightarrow \frac{1}{2}\frac{y}{x} - \log\frac{y}{x} = 2\log x - 2\log c$

$\Rightarrow \frac{y}{x} - \log y + \log x = 2\log x - \log k$

$\Rightarrow \frac{y}{x} = \log x + \log y - \log k$

$\Rightarrow \frac{y}{x} = \log\frac{xy}{k}$

$\Rightarrow \frac{xy}{k} = e^{y/x}$

$\Rightarrow xy = k\cdot e^{y/x}$ Ans.

$\Rightarrow \frac{1}{2}\int dv - \frac{1}{2}\int \frac{dv}{v} = \int dx$

$\Rightarrow \frac{1}{2}v - \frac{1}{2}\log v = 2x$

$\Rightarrow \frac{y}{2x} - \log\frac{y}{x} = 2x$

$\Rightarrow \frac{y}{x} - 2x = \log\frac{y}{x}$

11. $\frac{dy}{dx} = \frac{y(x-2y)}{x(x-3y)}$

Put $y = vx \Rightarrow v = y/x$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{vx(x-2vx)}{x(x-3vx)} = \frac{v(1-2v)}{1-3v}$

$\Rightarrow x\frac{dv}{dx} = \frac{v(1-2v)}{1-3v} - v = \frac{v-2v^2-v+3v^2}{1-3v} = \frac{v^2}{1-3v}$

$\Rightarrow \frac{1-3v}{v^2}\cdot dv = \frac{dx}{x}$

$\Rightarrow \int \frac{dv}{v^2} - 3\int \frac{dv}{v} = \int \frac{dx}{x}$

$\Rightarrow \frac{-1}{v} - 3\log v = \log x$

$\Rightarrow \frac{-x}{y} - 3\log\left(\frac{y}{x}\right) = \log x$

$\Rightarrow \frac{-x}{y} - 3\log y + 3\log x = \log x$

$\Rightarrow \frac{-x}{y} - 3\log y + 2\log x + \log c = 0$

$\Rightarrow \frac{-x}{y} + \log\frac{cx^2}{y^3} = 0$

$\Rightarrow \log\frac{cx^2}{y^3} = \frac{x}{y}$

$\Rightarrow \frac{cx^2}{y^3} = e^{x/y} \Rightarrow y^3 \cdot e^{x/y} = cx^2$ A